September 2001
IUT-Phys/01-11

Constraint structure in modified Faddeev-Jackiw method

A. Shirzad [1] , M. Mojiri [2]

*Institute for Studies in Theoretical Physics and Mathematics
P. O. Box: 5746, Tehran, 19395, IRAN,
Department of Physics, Isfahan University of Technology
Isfahan, IRAN.*

Abstract

We show that in modified Faddeev-Jackiw formalism, first and second class constraints appear at each level, and the whole constraint structure is in exact correspondence with level by level method of Dirac formalism.

[1]e-mail: shirzad@theory.ipm.ac.ir
[2]e-mail:mojiri@sepahan.iut.ac.ir

The standard method for analysis of the constrained systems is known to be the Dirac formalism [1]. Faddeev and Jackiw [2], however, have proposed an alternative method (FJ formalism). Their approach is based on solving the constraints at each level, and using the Darboux theorem [3] to find a smaller phase space together with a number of additional coordinates. The equations of motion of these additional coordinates will survive for new constraints, and the procedure will be repeated.

In a newer approach to FJ formalism [4, 5], called *symplectic analysis* or *modified Faddeev-Jackiw* (MFJ) formalism, instead of solving the constraints, one adds their time derivatives to the Lagrangian and considers the corresponding Lagrange multipliers as additional coordinates. In this way, constraints would be introduced in the kinetic part of the Lagrangian, rather than the potential. There are some efforts to show the equivalence of FJ or MFJ method with Dirac method. In [6], using a special representation of constrains, it is shown that principally first and second class constraints do appear at a typical step of FJ formalism. However, within the Dirac formalism there exists a well-established constraint structure, such that at each level of consistency, constraints divide to first and second class ones [8]. Then the consistency of the second class constraints determines some of Lagrange multipliers, while the consistency of first class ones leads to constraints of the next level.

Such a constraint structure is not known in FJ or FJM formalism. We show in this paper that the constraint structure of MFJ formalism emerges in the same way as in Dirac formalism. However, this structure, which is exactly similar to that given in [8], is somehow hidden within the Darboux theorem or within general statements. It should be noted that some signals of this structure has been recognized in [7], but that belong to cases where only first or second class constraints are present.

Suppose we are given the first order Lagrangian

$$L = a_i(y)\dot{y}^i - H(y) \tag{1}$$

in which y^i are coordinates not nessesarily canonical of a $2N$ dimensional phase space. The equations of motion read

$$f_{ij}\dot{y}^j = \partial_i H \tag{2}$$

where ∂_i means $\partial/\partial y^i$ and

$$f_{ij} \equiv \partial_i a_j(y) - \partial_j a_i(y). \tag{3}$$

The tensor f_{ij} is called the *Presymplectic* tensor. We denote it in matrix notation as f. If y^i are chosen to be the usual canonical variables (q, p) then the term $a_i(y)\dot{y}^i$ in (1) would be $p_i\dot{q}^i$. In other words, in some general (non canonical) coordinates y^i of the phase space, the term $a_i(y)\dot{y}^i$ has the same role as $p_i\dot{q}^i$. Therefore, according to construction of first order Lagrangian, it is reasonable that f is not singular. In fact the constraints would be imposed thereafter to a Lagrangian with nonsingular presymplectic tensor, as we will see. However, if for some artificial Lagrangian, f is singular with null eigenvectors $v_a^i(y)$, then constrains $\Phi_a(y) = v_a^i(y)\partial_i H(y)$ would result from equations of motion (2). In this case, one can treat with the constraints in the same manner as we will do in the following, and there in no serious difference. So, from now on we assume that the presymplectic tensor f is not singular. Let denote components of f^{-1} as f^{ij}. Then equations (2) read

$$\dot{y}^i = f^{ij}\partial_j H. \tag{4}$$

The antisymmetric tensor f^{ij} defines the Poisson brackets of phase space coordinates:

$$\left\{y^i, y^j\right\} = f^{ij}. \tag{5}$$

Assuming that

$$\{F(y), G(y)\} = \partial_i F \partial_j G \left\{y^i, y^j\right\} \tag{6}$$

the Poisson bracket of two arbitrary functions $F(y), G(y)$ can be obtained as

$$\{F(y), G(y)\} = \partial_i F f^{ij} \partial_j G. \tag{7}$$

Suppose we want to impose the primary constraints $\Phi_\mu^{(0)}(\mu = 1, ..., M)$ to the system given by Lagrangian (1). This can be done by considering the Lagrangian

$$L' = a_i(y)\dot{y}^i - H(y) - v^\mu \Phi_\mu^{(0)} \tag{8}$$

with v_μ as additional coordinates. Hence the presymplectic tensor f should be replaced by the $(2N + M) \times (2N + M)$ singular tensor

$$f' = \left(\begin{array}{c|c} f & 0 \\ \hline 0 & 0 \end{array}\right). \tag{9}$$

Multiplying both sides of (2) with the null eigenvevtors of (9) restores the constraints $\Phi_\mu^{(0)}$. Therefore the additional term $v^\mu \Phi_\mu^{(0)}$ in (8) has no new result. However, consistency of the constraints during time imposes new constraints on system, i.e. $d\Phi^0_\mu/dt$ should vanish. Adding the term $\eta^\mu \dot{\Phi}^0_\mu$ to L, the modified Lagrangian would be

$$L^{(1)} = (a_i - \eta^\mu A_{\mu i})\, \dot{y}^i - H(y), \tag{10}$$

where

$$A_{\mu i} \equiv \partial_i \Phi^{(0)}_\mu . \tag{11}$$

It should be noted that , as shown in [6], solving the constraints and considering the equations of motion for the non canonical coordinates are equivalent to considering the consistency of constraints $\Phi^{(0)}_\mu$.

Considering $Y \equiv (y^i, \eta^\mu)$ as coordinates, the symplectic tensor $f^{(1)}$ reads

$$f^{(1)} = \left(\begin{array}{c|c} f & A \\ \hline -\tilde{A} & 0 \end{array} \right) \tag{12}$$

where the elements of $(2N \times M)$ matrix A are defined in (11) and $\tilde{A}$ is the transposed matrix. The equations of motion in matrix notation then reads

$$f^{(1)} \dot{Y} = \partial H. \tag{13}$$

Using operations that keep the determinant invariant, it is easy to show that

$$\begin{aligned} \det f^{(1)} &= \det \left(\begin{array}{c|c} f & A \\ \hline 0 & \tilde{A} f^{-1} A \end{array} \right) \\ &= (\det f)(\det \tilde{A} f^{-1} A). \end{aligned} \tag{14}$$

Assuming that $\det f \neq 0$, $f^{(1)}$ would be singular if $C \equiv \tilde{A} f^{-1} A$ is singular. Using (5) and (11) we have

$$C_{\mu\nu} = \left\{ \Phi^{(0)}_\mu , \Phi^{(0)}_\nu \right\}. \tag{15}$$

Therefore, the singularity disappears if $C_{\mu\nu}$ is invertible (this is the case considered in [7]). In Dirac terminology this means that all primary constrains are second class. In this case ${f^{(1)}}^{-1}$ defines a new bracket in the same way as in (7). It can be shown that:

$${f^{(1)}}^{-1} = \left(\begin{array}{c|c} f^{-1} - f^{-1} A C^{-1} \tilde{A} f^{-1} & -f^{-1} A C^{-1} \\ \hline C^{-1} \tilde{A} f^{-1} & C^{-1} \end{array} \right). \tag{16}$$

Using (16) one can easily see that the new bracket between two functions of the original phase space is the well-known Dirac bracket:

$$\{F,G\}_{D.B} \;=\; \{F,G\} - \left\{F,\Phi_{\mu}^{(0)}\right\} C^{\mu\nu} \left\{\Phi_{\nu}^{(0)},G\right\} \tag{17}$$

where $C^{\mu\nu}$ is the inverse of $C_{\mu\nu}$.

Now suppose that $C_{\mu\nu} = \left\{\Phi_{\mu}^{(0)},\Phi_{\nu}^{(0)}\right\}$ is singular. As in the framework of Dirac formalism [8], we can assume that the constraints $\Phi_{\mu}^{(0)}$ divide to M' first and M'' second class constraints $\Phi_{\mu'}^{(0)}$ and $\Phi_{\mu''}^{(0)}$ respectively $(M'+M''=M)$ with the following property

$$\begin{array}{l} \left\{\Phi_{\mu'}^{(0)},\Phi_{\nu'}^{(0)}\right\} \approx 0 \\ \left\{\Phi_{\mu'}^{(0)},\Phi_{\nu''}^{(0)}\right\} \approx 0 \\ \left\{\Phi_{\mu''}^{(0)},\Phi_{\nu''}^{(0)}\right\} \approx C_{\mu''\nu''} \qquad \det C_{\mu''\nu''} \neq 0 \end{array} \tag{18}$$

where $\approx$ means weak equality, i.e. equality on the surface of constraints known already (here, Primary constraints). Defining

$$\begin{array}{l} A_{\mu' i} = \partial_i \Phi_{\mu'}^{(0)} \\ A_{\mu'' i} = \partial_i \Phi_{\mu''}^{(0)} \end{array} \tag{19}$$

the tensor $f^{(1)}$ reads

$$f^{(1)} = \left(\begin{array}{c|c|c} f & A'' & A' \\ \hline -\tilde{A}'' & 0 & 0 \\ \hline -\tilde{A}' & 0 & 0 \end{array}\right). \tag{20}$$

Using (18) and (15) it is easy to show that the rows of

$$\left(\tilde{A}' f^{(-1)}, 0, 1\right) \tag{21}$$

which is an $M' \times (2N + M'' + M')$ matrix, are null eigenvectors of $f^{(1)}$. Multiplying both sides of (13) with each row of (21), say row μ', gives the new constraint

$$\Phi_{\mu'}^{(1)} \equiv \left\{\Phi_{\mu'}^{(0)}, H\right\} = 0. \tag{22}$$

This is exactly the same result as obtained in Dirac formalism [8], i.e. the consistency of first calls constraints at each level gives the constraints of the next level.

On the other hand, note that the $f^{(1)}$ matrix in (20) has an invertible sub-block

$$f^{(1)}_{\rm inv} = \left(\begin{array}{c|c} f & A'' \\ \hline -\tilde{A}'' & 0 \end{array}\right) \tag{23}$$

The inverse of (23) is similar to (16). Dividing Lagrange multipliers η^{μ} to $\eta^{\mu'}$ and $\eta^{\mu''}$ corresponding to first and second class constraint respectively, one can use the inverse of (23) to find $\dot{\eta}^{\mu''}$ from (13). The result is

$$\dot{\eta}^{\mu''} = -C^{\mu''\nu''}\left\{\Phi^{(0)}_{\nu''}, H\right\}, \tag{24}$$

where $C^{\mu''\nu''}$ is the inverse of

$$C_{\mu''\nu''} = \left\{\Phi^{(0)}_{\mu''}, \Phi^{(0)}_{\nu''}\right\}. \tag{25}$$

Adding a total derivative, one can replace the term $\eta^{\mu''}\dot{\Phi}^{(0)}_{\mu''}$ in Lagrangian (10) with $-\dot{\eta}^{\mu''}\Phi^{(0)}_{\mu''}$. Then from (24), one observes that the replacement

$$H^{(1)} = H^{(0)} - \left\{H^{(0)}, \Phi^{(0)}_{\mu''}\right\} C^{\mu''\nu''}\Phi^{(0)}_{\nu''} \tag{26}$$

suffices to omit second class constraints $\Phi^{(0)}_{\mu''}$ in the remaining and to work only with first calls constraints $\Phi^{(0)}_{\mu'}$. Hence, before going to next level, we assume that our Lagrangian is

$$L^{(1)} = \left(a_i - \eta^{(0)\mu}\frac{\partial\Phi^{(0)}_{\mu}}{\partial y^i}\right)\dot{y}^i - H^{(1)}(y) \tag{27}$$

where all primary constraints $\Phi^{(0)}_{\mu}$ are first class. (These are in fact the previous $\Phi^{(0)}_{\mu'}$, where for simplicity we have omitted the primes.) The corresponding symplectic tensor reads

$$f^{(1)} = \left(\begin{array}{c|c} f & A^{(0)} \\ \hline -\tilde{A}^{(0)} & 0 \end{array}\right) \tag{28}$$

where $A^{(0)} \equiv \partial_i\Phi^{(0)}_{\mu}$ stands for gradient of primary constraints which, as stated, are assumed to be first class so far.

Now let's go to the level step. This aim would be achieved by adding the consistency term $-\eta^{(1)\mu}\dot{\Phi}^{(0)}_{\mu}$ to the Lagrangian (27) to give

$$L^{(2)} = \left(a_i - \eta^{(0)\mu}\frac{\partial\Phi^{(0)}_{\mu}}{\partial y^i} - \eta^{(1)\mu}\frac{\partial\Phi^{(1)}_{\mu}}{\partial y^i}\right)\dot{y}^i - H^{(1)}(y) \tag{29}$$

Taking $\eta^{(1)}_{\mu}$ as new variables, we have

$$f^{(2)} = \left(\begin{array}{c|c|c} f & A^{(0)} & A^{(1)} \\ \hline -\tilde{A}^{(0)} & 0 & 0 \\ \hline -(\tilde{A})^{(1)} & 0 & 0 \end{array}\right) \tag{30}$$

As we will see (for an arbitrary level), $\det f^{(2)}$ vanishes if the matrix $C^{(1)}_{\mu\nu} = \left\{\Phi^{(0)}_{\mu}, \Phi^{(1)}_{\nu}\right\}$ is singular, and vise versa. If $\det C^{(1)} \neq 0$ then the singularity disappears at all. This means that the system is completely second class. If, however, $C^{(1)}$ is singular, then one can in principle divide $\Phi^{(1)}_{\mu}$ to $\Phi^{(1)}_{\mu'}$ and $\Phi^{(1)}_{\mu''}$, and $\Phi^{(0)}_{\mu}$ to $\Phi^{(0)}_{\mu'}$ and $\Phi^{(0)}_{\mu''}$ in such a way that

$$\begin{array}{l} \left\{\Phi^{(1)}_{\mu'}, \Phi^{(0)}_{\mu}\right\} \approx 0 \\ \left\{\Phi^{(1)}_{\mu''}, \Phi^{(0)}_{\nu''}\right\} \approx C^{(1)}_{\mu''\nu''} \qquad \det C^{(1)}_{\mu''\nu''} \neq 0 \end{array} \tag{31}$$

Rearranging $f^{(2)}$ one can find the invartible sub-block:

$$f^{(2)}_{\text{inv}} = \left(\begin{array}{c|c|c} f & A^{(0)\prime\prime} & A^{(1)\prime\prime} \\ \hline -\tilde{A}^{(0)\prime\prime} & 0 & 0 \\ \hline -\tilde{A}^{(1)\prime\prime} & 0 & 0 \end{array}\right). \tag{32}$$

Using inverse of (32), $\dot{\eta}^{(0)}_{\mu''}$ and $\dot{\eta}^{(1)}_{\mu''}$ can be derived from the equation of motion (13). This leads to defining a newer Hamiltonian $H^{(2)}$ whose explicit form is not important. Moreover, multiplying the equations of motion with the null eigenvectors of

$$\left(\tilde{A}^{(1)\prime} f^{(-1)}, 0, 0, 0, 1\right) \tag{33}$$

gives the third level constraints

$$\Phi^{(2)}_{\mu'} = \left\{\Phi^{(1)}_{\mu'}, H^{(1)}\right\}. \tag{34}$$

To complete the discussion, and see some more details, let us consider the n-th step.

Suppose

$$\Phi_\mu^{(n)} = \left\{ \Phi_\mu^{(n-1)}, H^{(n-1)} \right\} \tag{35}$$

are n-th level constraints;

$$L^{(n)} = \left(a_i - \sum_{k=1}^{n-1} \eta^{(k)\mu} \frac{\partial \Phi_\mu^{(k)}}{\partial y^i} \right) \dot{y}^i - H^n(y) \tag{36}$$

is the n-th level Lagrangian and

$$f^{(n)} = \left(\begin{array}{c|ccc} f & A^{(0)} & \cdots & A^{(n)} \\ \hline -\tilde{A}^{(0)} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -\tilde{A}^{(n)} & 0 & \cdots & 0 \end{array} \right) \tag{37}$$

is the n-th level symplectic tensor, where

$$A_{\mu i}^k = \partial_i \Phi_\mu^{(k)}. \tag{38}$$

Omitting second class constraints up to level $(n-1)$, we have assumed that constraints $\Phi_\mu^{(0)}, \Phi_\mu^{(1)} \cdots \Phi_\mu^{(n-1)}$ are first class. Using operations that leave the determinant invariant, one can show that

$$\det f^{(n)} = (\det f).\det \begin{pmatrix} P^{01} & \cdots & P^{0n} \\ \vdots & & \vdots \\ P^{n0} & \cdots & P^{nn} \end{pmatrix} \tag{39}$$

where

$$P^{kl} = \tilde{A}^k f^{(-1)} A^l = \left\{ \Phi^{(k)}, \Phi^{(l)} \right\}. \tag{40}$$

Using Jacobi identity and the recursion relation

$$\Phi_\mu^{(k)} \approx \left\{ \Phi_\mu^{(k-1)}, H \right\} \tag{41}$$

it is easy to show

$$\left\{ \Phi_\mu^{(k)}, \Phi_\nu^{(l)} \right\} \approx - \left\{ \Phi_\mu^{(k-1)}, \Phi_\nu^{(l+1)} \right\}. \tag{42}$$

Since $\Phi_\mu^{(0)}, \Phi_\mu^{(1)} \cdots \Phi_\mu^{(n-1)}$ are assumed to be first class, using (42) we observe that

$$P^{kl} \approx 0 \quad \text{if} \qquad k + l < n \tag{43}$$

In other words the matrix P has the following form

$$P = \begin{pmatrix} 0 & 0 & \cdots & 0 & P^{0n} \\ 0 & 0 & \cdots & P^{1(n-1)} & P^{1n} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & P^{(n-1)1} & \cdots & P^{(n-1)(n-1)} & P^{(n-1)n} \\ P^{n0} & P^{n1} & \cdots & P^{n(n-1)} & P^{nn} \end{pmatrix}. \tag{44}$$

Then using (42) we can write

$$\det P \propto \left(\det P^{0n}\right)^n. \tag{45}$$

Therefore, $\det f^{(n)}$ vanishes if $\left\{\Phi_\mu^{(0)}, \Phi_\nu^{(n)}\right\}$ is singular. This is also an important observation in Dirac theory [8], that the criterion to distinguish between first and second class constraints is singularity of their Poisson brackets with primary constraints. If $\det P^{0n} \neq 0$ then the singularity of the theory disappears and all of the constraints would be second class. If, however, P^{0n} is singular, one should divide all sets of constraints to first class ones: $\Phi_{\mu'}^{(0)}, \cdots \Phi_{\mu'}^{(n)}$ and second class ones: $\Phi_{\mu''}^{(0)}, \cdots \Phi_{\mu''}^{(n)}$. The matrices $A^{(0)}, \cdots A^{(n)}$ also break into $A^{(0)\prime}, \cdots A^{(n)\prime}$ and $A^{(0)\prime\prime}, \cdots A^{(n)\prime\prime}$ accordingly. Rearranging $f^{(n)}$ such that $A^{(0)\prime\prime}, \cdots A^{(n)\prime\prime}$ come first, we will find the invertible sub-block as

$$f_{\text{inv}}^{(n)} = \left(\begin{array}{c|ccc} f & A^{(0)\prime\prime} & \cdots & A^{(n)\prime\prime} \\ \hline -\tilde{A}^{(0),\prime\prime} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -\tilde{A}^{(n)\prime\prime} & 0 & \cdots & 0 \end{array}\right) \tag{46}$$

The inverse can be found to be

$$f_{\text{inv}}^{(n)^{-1}} = \left(\begin{array}{c|c} f^{-1} - f^{-1}AC^{-1}\tilde{A}f^{-1} & -f^{-1}AC^{-1} \\ \hline C^{-1}\tilde{A}f^{-1} & C^{-1} \end{array}\right) \tag{47}$$

where A denotes the matrix:

$$A = \left(\begin{array}{c|c|cc} A^{(0)\prime\prime} & A^{(1)\prime\prime} & \cdots & A^{(n)\prime\prime} \end{array} \right). \tag{48}$$

This enables us to solve the equations of motion to find $\dot{\eta}_\mu^{(0)\prime\prime}, \cdots \dot{\eta}_\mu^{(n)\prime\prime}$. Substituting into Lagrangin, the modified Hamiltonian $H^{(n+1)}$ would be derived. Finally the null-eigenvectors of $f^{(n)}$ (after rearranging) can be written as

$$(\tilde{A}^{(n),\prime} f^{(-1)}, \overbrace{0, 0, \cdots, 0}^{2n-1}, 1). \tag{49}$$

Multiplying the equations of motion with (49) finally gives the constraints of the next level

$$\Phi_\mu^{(n+1)} = \left\{\Phi_\mu^{(n)}, H^{(n)}\right\} \tag{50}$$

As is observed, the whole thing goes on in MFJ formalism in the same way as in Dirac formalism, i. e. at each level a number of second class constraints $(\Phi_{\mu''}^{(0)}, \cdots, \Phi_{\mu''}^{(n)})$ are separated, and a set of new constraints $\Phi_\mu^{(n+1)}$ emerge. The whole procedure will finish at level N, say, in two case: first, if $\Phi_\mu^{(N+1)} \approx \left\{\Phi_\mu^{(N)}, H\right\}$ vanish weakly and next, if the symplectic matrix $f^{(N)}$ is invertible. In the first case the system possesses gauge invariance generated by $(\Phi_\mu^{(0)}, \cdots, \Phi_\mu^{(N)})$ (assuming that we have put away second class constraints in previous levels). In the second case the system is completely second class and there is no gauge invariance.

As a final point, we give a few words about a seemingly contradiction between Dirac and MFJ methods appeared in [5] when discussing the model

$$L = (q_2 + q_3)\,\dot{q}_1 + q_4 \dot{q}_3 + \frac{1}{2}\left(q_4^2 - 2q_2 q_3 - q_3^2\right). \tag{51}$$

This model was first introduced in [10] as an example that the equation of motion can not be derived from a second order Lagrangian. Then considering q_i as coordinates of a configuration space, [11] has discussed constraint structure of the model in Dirac theory. In this way four second class constraints would emerge and the dynamics on the constraint surface goes on via Dirac brackets. The model then is analysed in [5] using MFJ method and no constraint has been encountered.

To see what has happened, we consider the general form of a first order Lagrangian as given by (1). We emphasize that in FJ method the variables of the first order Lagrangian are viewed as coordinates of a phase space. It is, however, possible to consider those variables as coordinates of a configuration space. Hence, suppose we are given the Lagrangian

$$L = a_i(q)\dot{q}_i - V(q) \tag{52}$$

which is obviously singular. Following Dirac method, the primary constraints are

$$\Phi_i = p_i - a_i(q) \approx 0 \tag{53}$$

We see that dublicating the space of q^i to phase space of (q^i, p_i) is compensated by the same number of constraints Φ_i.

Since there is no quadratic term in Lagrangian (52), the canonical Hamiltonian is the same as $V(q)$ and the total Hamiltonian reads

$$H_T = V(q) + \lambda^i \left(p_i - a_i(q)\right) \tag{54}$$

where λ^i are Lagrangian multipliers. It is easy to see that

$$\{\Phi_i, \Phi_j\} = \frac{\partial a_j}{\partial q^i} - \frac{\partial a_i}{\partial q^j} \equiv f_{ij}. \tag{55}$$

Suppose f is not singular. In the context of MFJ method, this leads to solving equations of motion (2) to get

$$\dot{q}^i = f^{ij} \partial_j V. \tag{56}$$

However, in the context of Dirac theory, nonsingularity of f means that the constraints Φ_i are second class and consequently the constraint surface given by $p_i = a_i(q)$ has a phase space structure by itself.

The consistency conditions $\dot{\Phi}_i \approx 0$ gives

$$\{\Phi_i, H_T\} = -\partial_i V + f_{ij} \lambda^j \tag{57}$$

which determines all λ^j in terms of coordinates. Inserting in (54) gives

$$H_T = V(q) + \left(p_i - a_i(q)\right) f^{ij} \partial_j V \tag{58}$$

The canonical equations of motion for q^i give

$$\dot{q}^i = \frac{\partial H_T}{\partial p_i} = f^{ij} \partial_j V; \tag{59}$$

the same result as (56). (The equation $\dot{p}_i = -\frac{\partial H_T}{\partial q^i}$ give time derivative of (53), which is obvious.)

We see that for a regular first order Lagrangian the Dirac procedure (after turning around) gives the same result as one can obtain directly form MFJ formalism. It is not difficult to see that the same result is correct if the system is singular. So the apparent contradiction between Dirac and FJ methods is not in fact a contradiction. The basic point is considering y^i (in first order Lagrangian) as phase space or configuration space coordinates.

Concluding, we think that FJ and MFJ formalism are basically the same as Dirac formalism formulated in the language of first order Lagrangians. From another point of view our work here was based on extending the symplectic tensor at each step and studying it's singularity properties. This, however, is closely similar to method given in [9], where for a second or higher order Lagrangian the Hessian matrix is extended at each step and it's null eigenvectors are searched for.

Acknowledgement

We thank F. Loran for useful discussions.

References

[1] P. A. M. Dirac, *"Lectures on Quantum Mechanics"* New York: Yeshiva University Press, 1964

[2] L. Faddeev and R. Jackiw ,Phys. Rev. Lett. 60, 1692(1988)

[3] P. J. Olver, Application of lie groups to differential equation, Graduate texes in mathematics 107, Springer Verlag (New York, 1986)

[4] J.Barcelos-Neto and C. Wotzasek, Mod. Phys. Lett. A7,1737(1992); Int. J. Mod. Phys. A7, 4981(1992)

[5] J.Barcelos-Neto and N. R. F. Beraga, J. Math. Phys., Vol. 35, No. 7, 3497(1994)

[6] J. Antonio Garcia and J. M. Pons, Int. J. Mod. Phys. A12, 451(1997)

[7] H. Montani Int. J. Mod. Phys. A8, 4319(1993)

[8] C. Batlle, J. M. Pons, N. Roman-Roy, J. Math. Phys. 27, 2953(1986)

[9] A. Shirzad, J. Phys. A: Math. Gen. 31, 2747(1998)

[10] S. Hojman and L. C. Shepley, J. Math. Phys. 32, 142(1991)

[11] U. Kulshreshtha, J. Math. Phys. 33, 633(1992)